An Interactive Evaluation System for Learning Mathematics

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We present an interactive evaluation system for mathematics in the e-learning Moodle environment. One of the most important aspects of the teaching-learning process is to check the acquired knowledge and to achieve this goal we submit the students to questions, tests, and exams in their course materials. In this article we present a system which makes possible to choose a subject from a given list, in some area of mathematics, and the tool generates a test or exam for that subject. The kind of questions which are available are not only the usual multiple-choice questions but it is also possible to give short-answer questions. The system has algorithmic variations features and automatically generates slightly different instances of a test or exam, in such a way that a student would not get the same thing twice. From teachers' side it is possible to give complete answers and detailed solutions to the questions and record scores. Based on my experience in teaching mathematics for mathematicians and engineers it provides questions that can check the calculating ability of a student but also the knowledge of a subject including proofs of theorems.

Keywords E-learning; Information and Communication Technologies; Mathematics; Latex

1. Introduction

One of the main keys for the future of technologies is mathematics. Basic mathematics has always been an inherent and integral part of the human knowledge and in higher levels is very important for many fields of research. The fruitful interaction between mathematics and other sciences bring benefits for both and these interactions sometimes led to the development of entirely new disciplines. Although mathematical ideas exist independently of the symbols and notations used we always need symbolic forms to represent and manipulate these ideas, so one of the challenges in developing e-learning strategies for mathematics is to have technologies which permit their integration in the same environment. Due to limitations of HTML and in order to meet the diverse needs of the scientific community extensible markup language (XML) and the XML-application (application of the XML to a specific domain, e.g. mathematics, physics, chemistry, etc.) mathematics markup language (MathML) defined by the World Wide Web Consortium (W3C) are important steps in developing mathematics on the Web (see [1]).

The importance of giving to the next generation the skills they need is far beyond the traditional methods (paper and pen) of teaching in today's universities. The students' assessment using the new technologies is one of the centrepieces of many educational improvements efforts in this direction. (see [2]) Therefore, one of the major tasks of the mathematical education it will be the development of the teaching-learning process through these technologies.

2. Description

There are entire books devoted to the conventions of mathematical typesetting, from the alignment of subscripts and superscripts to rules for choosing parenthesis sizes. Most of mathematicians and scientists

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usually write mathematical text using a Tex/Latex system. The Tex system is a sophisticated program created by Donald Knuth, which was designed to produce high-quality typesetting, especially for mathematical text. This system has become very popular in the scientific community because it can be used for transforming any kind of writing into many types of documents (e.g. articles, books, reports, etc.) in a way that can be specified completely by the writer through a rich language of commands (these commands are written with a back slash followed by the command itself, e.g. \begin{document}). It is a portable system, running on a wide range of computer platforms and it is also available as public domain. Based on the Tex system, Leslie Lamport started to work on a document preparation system which adds to Tex a collection of macros that simplify typesetting by letting the user concentrate on the structure of the text rather than on the formatting commands. A Latex file is a plain text file (which contains ordinary text and the commands, i.e. the literal text and the markup language), which has the extension .tex and it is usually prepared with a text editor or word processor. The output from a .tex file through one of the Tex systems is a set of files, and one of the most important is one with the extension .dvi, which contains a representation of the formatted text where the type and position on the page are specified for each character to be outputted. We also need a dvi driver to produce further output designed specifically for a particular printer or to be used as input to a previewer for display on a computer screen and visualize the result (for more details in Latex language see [3-4]).

3. Main aspects

Some of the text editors or word processors that we use to write text are very simple (like Notepad or WordPad which are provided with the operating system Windows) while others offer a broad and complex range of functionalities (e.g. syntax highlighting and automatic completion, like WinEdt). It is important to clarify the distinction between the Tex system itself and a text editor or a word processor that we use to write text. We always need a Tex system (e.g., PlainTex, emTex, MikTex, PcTex, TexLive, TrueTex, etc.) to compile a tex file, but the text editor or the word processor are just ways to simplify the process of writing in the markup language. Most of the work we present here is based on the software Scientific Workplace (SWP) from MacKichan Software, Inc, which is a Latex-based word processor that completely integrates text and mathematics in the same environment. The program has also a link to a Computer Algebra System (CAS) the MuPAD engine (from SciFace software GmbH), which gives a great flexibility to the program in terms of numerical and symbolic computations. A complete integration of this software in a Web site platform (e.g. Moodle environment) is one of our goals.

3.1 File formats and templates

The process starts with a .tex file and this file is divided into major sections. These sections are the Section headings, which begin with keyword headings, and the Section paragraphs that contain keywords recognized by the tool, so in general a file begins with a keyword heading (usually written between the commands \begin{document} and \end{document}) identifying it as a test (or exam, homework, quiz, tutorial). It is always possible to add comments and text (or some kind of information that the user wants to include) which explains the purpose of the assessment and this is done by the keywords Comment and Text. The text written in the keyword Comment is not visible for the student. One of the most important sections is the main setup section, and in this section we place all the options that manage the overall generation of the instances, which means that every time we open a file we will obtain a different one. When we compile the file the software detects certain syntactic errors and then an instance will not be created and showed, instead of this we will see the original file marked up with error messages. Each instance created has at the end a button for the student grade the test. After the answers are sent to the grader the program displays the results and the time spent on the test in a new tex file. The table below shows the main page that we created for the course of e-algebra:

e-Algebra

(main page)

An interactive course in Algebra

	Disciplines:	Hyperlinks:
1)	Linear Algebra I	Test LA-I
2)	Linear Algebra II	Test LA-II
3)	Algebra I	Test A-I
4)	Algebra II	Test A-II

3.2 Questions

We are interested in verifying the knowledge and skills of a student. This can be done not only with the traditional multiple-choice questions (in particular, true-false), which basically test facts and skills in isolation (e.g., the calculation ability of a student) and does not really check the concept which is behind a mathematical notion (sometimes needed in real-life situations), but it is also possible to prepare free-response questions. It is also possible to give detailed solutions of the questions or to guide a student through the main steps of an exercise. Another possibility is to guide the student through theoretical concepts, e.g. a definition or a proof of a theorem.

There are many ways to construct the questions and the type of questions which are possible to create are:

- Multiple-choice questions (in particular, true-false);
- Questions with hyperlinks;
- Free-response questions;
- Filled out questions;
- Questions with a special form-response that can be automatically evaluated.

One of the potentials of the software lies in the automatic generation of slightly different instances of course materials based on the same file. It is possible to build algorithmic variations in several ways:

- Permute the order of the questions and inside of every question the order of the items;
- For each question we can provide several equivalent questions (variants) and randomly select one or more for inclusion in the instance;
- Vary the questions themselves by creating questions and answers using random numbers and random matrices;
- Insertion of random variables and formulas and the possibility to impose conditions;
- Insertion of plots generated by random functions.

3.3 Examples

When we prepare a question, related with the process of construction of the question itself there is another process of elaboration, this means that we can not just simply write down the question and say, it is done. Sometimes we need to think of what kind of formulas and conditions we want to impose, for the question makes sense. A formula is a special type of a mathematical expression that is evaluated before it is displayed on the screen. For example, if we enter a formula consisting of the product of two quadratic polynomials $(a+bx+cx^2)(d+ex+fx^2)$, the program displays the result of this product on the screen (the quartic polynomial). When we use the generation of random numbers, the results can at times be unpredictable, and to prevent such results we specify conditions that randomly defined variables must meet. For example, we might set a condition that prevents a variable from being defined as some number or a condition that prevents one variable from being smaller (or greater) than other. A question that involves two randomly generated variables x and y might have as possible multiple-choice answers both x / y and y / x. If the randomly generated values for the variables are the same, these answers will be identical and will present a confusing choice to the student. We can avoid this by setting the conditions $x \neq y$ and also want to avoid the situation of $x = 0$ and $y = 0$. These conditions can be combined with boolean operators and "$\wedge$" (resp., or "$\vee$") and with binary relations $\geq$ "greater or equal than" (resp., $\leq$ "less or equal than").

To make these things clear I will try to explain in more detail some examples:

As a first example, suppose we want that a student factors a quartic polynomial and we want that the polynomial can be expressed as the product of two quadratic polynomials. So, the question can be "Find the roots of the quartic polynomial $ax^4+bx^3+cx^2+dx+e$". If we want the letters *a*, *b*, *c*, *d*, and *e* vary randomly (e.g. integer numbers between -4 and 4) then we have to be careful, since we don't want *a*, *b*, *c*, *d*, and *e* to be completely arbitrary, since not all possibilities produced randomly can be factorized. For example, if $a = 0$ we do not have a quartic polynomial, and the question does not make sense. Another thing is that we want the polynomial can be factorized into quadratic polynomials, so we need to introduce the result as a product of two quadratic polynomials $(a+bx+cx^2)(d+ex+fx^2)$, since in this way the polynomial is always factorizable, and this will be evaluated before the student sees the result on the screen, which with the conditions $c \neq 0$ and $f \neq 0$ guarantees that we have a quartic polynomial. Now suppose we want to check if the student really knows how to use the formula to solve a quadratic polynomial, but we don't want imaginary roots. For this, we have to impose another condition $b^2\text{-}4ca \geq 0$ (resp., $e^2\text{-}4fd \geq 0$), for the polynomial $a+bx+cx^2$ (resp., $d+ex+fx^2$) factor nicely. For the ones who like some code see Fig. 1:

Fig. 1: **Part of the code for the first example.**

$a := \text{rand}(\{-4,-3,-2,-1,1,2,3,4\})$

$b := \text{rand}(-4,4)$

$c := \text{rand}(-3,3)$

$d := \text{rand}(-4,4)$

$e := \text{rand}(-4,4)$

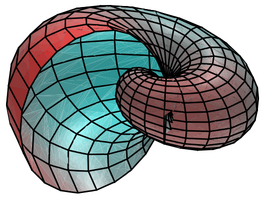
$f := \text{rand}(-3,3)$

$h := (a+bx+cx^2)(d+ex+fx^2)$

Conditions: $(c \neq 0 \wedge f \neq 0) \wedge (c \neq 1 \vee f \neq 1) \wedge (b^2 - 4ca \geq 0) \wedge (e^2 - 4fd \geq 0)$

Another example is the following:
Suppose we want to plot the function $f(x,y)=(1.2)^x\sin(y)$, where *x* and *y* are random numbers belonging to some interval. We can put the following multiple-choice question:

Statement:
Calculate the values of x and y in such a way that the function $f(x,y)=(1.2)^x\sin(y)$ has the following plot:



Choices:

a) $-1\leq x\leq 6.2$ and $0\leq y\leq 3.1$.
b) $0\leq x\leq 6.2$ and $0\leq y\leq 3.1$.
c) $-1\leq x\leq 6.2$ and $-1\leq y\leq 3.1$.
d) $0\leq x\leq 6.2$ and $-1\leq y\leq 3.1$.

4. Remarks and conclusions

One of the main advantages of the proposed system in this paper is to provide an integrated environment where a Tex/Latex and CAS systems are joined together. There is no doubt that we are only scratching the surface of these technologies and there is a long way to go through. We are conscious that for many institutions it will take time to implement and see the real value of this.

In our opinion, a good e-ducation will change the traditional teaching-learning processes as they are today and the educational institutions should take this leadership because courses through this kind of technologies will be a great challenge for the future.

References

[1] W3C Consortium, W3C Math Working Group, MathML. http://www.w3.org/Math.
[2] S. Jeschke, T. Richter, R. Seiler, Mathematics in Virtual Knowledge Spaces: User Adaptation by Intelligent Assistants, Proceedings of the 3rd International Conference on Multimedia and Information & Communication Technologies in Education, 7-10 June 2005, Cáceres, Spain.
[3] M. Goossens, F. Mittelbach, A. Samarin, The Latex Companion, Addison-Wesley, Reading, MA, 1994.
[4] M. Goossens, F. Mittelbach, S. Rahtz, The Latex Graphics Companion, Addison-Wesley, Reading, MA, 1997.
[5] A. Cos, J. Walls, S. Llinares, Interaction in Learning Environments and Learning to Teach Mathematics, Proceedings of the 3rd International Conference on Multimedia and Information & Communication Technologies in Education, 7-10 June 2005, Cáceres, Spain.
[6] J.L. Galán García et al., Computer Algebra Systems: A basic tool for teaching Mathematics in Engineering, Proceedings of the 3rd International Conference on Multimedia and Information & Communication Technologies in Education, 7-10 June 2005, Cáceres, Spain.